Regional topological segmentation based on Mutual Information Graphs

Ming Liu, Francis Colas, Roland Siegwart
Autonomous Systems Lab, ETH Zurich, Switzerland
[ming.liu, francis.colas]@mavt.ethz.ch, rsiegwart@ethz.ch

***Abstract*—When people communicate with the robots, the most direct mean is by naming the different regions in the environment. This capability of robot is highly depending on the unsupervised topological segmentation results. Nowadays lots of researches in this direction are based on the spectral clustering algorithm in graph theory. However there are inherent drawbacks of spectral clustering algorithms. In this paper, we first discuss these drawbacks using several testing results, then we propose our approach based on information theory in the formation of the graph structure, and uses Chow-Liu tree to segment the composed graph according to the weights' variations. The results show that our method provides a more flexible and rational result in the sense of aiding the semantic mapping or other further applications.**

I. Introduction

MAPPING is an environmental modelling method. The outcome of the mapping process is a representation of the surroundings namely a map. Usually, applicable maps can be categorized into metric maps, topological maps, hybrid maps and semantic maps etc. Specifically, the topological maps consist of nodes and edges between the nodes. The topological mapping techniques can be further categorized by what the nodes and edges represent. For the purpose of visual navigation, usually the term "nodes" represents the waypoints which the mobile robot should follow, and the "edges" are the physical transitions between nodes; on the other hand, for the reasoning of an environment or achieving a semantic representation of the environment, the "nodes" can represent functional regions, such as rooms. In this paper we will discuss the methods to create the topological maps which manifest the relations between different regions. In the rest of this paper, "topological map" indicates a representation of the relations between a number of regions, such that the "node" in the map represents a region of free space, and the edge represents the linkage between two regions.

The topological map is suitable for a better quality of human-robot interactions, comparing to the metric map. To a certain extend, topological mapping is the way used by human brains during our everyday life. Moreover, comparing to the metric maps, topological maps are more compact in the sense of storage saving, and scalability etc. Therefore, several works related to semantic mapping [1] [2] or cognitive mapping [3] used topological map as the basic representation of the environment. In this paper we will show that our method is able to generate a relatively stable topological segmentation of the map, which can be potentially used by the further environmental modelling.

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There are several representation method or descriptors using vision for the topological nodes, such as fingerprint of places [4] and adaptive color tags of places [5]. These descriptors can work under the condition that vision approach will provide nourishing raw information from the environment. As far as laser scan is concerned, the scan itself cannot provide as much direct information as vision. Therefore several methods treat a bunch of aligned consequent 2D scans as a sub-image, then perform the image-based segmentation based on vision techniques, such as [1] uses watershed to segment the given metric map. There are other approaches which uses a part of the occupancy grid map as a sub-map directly. This manner is considered as the most basic and efficient representation of a node, and got widely applied, such as [6] [7]. As for the methods in which the maps are constructed, several representation methods can be introduced [8], e.g. Voronoi diagram [9] and critical lines etc.

In this work we use structured cells, which are spanned in the global map, to represent the basic decomposition of the environment in a graph. Generally, the basis of the topological mapping is the topological segmentation, while the basis of the topological segmentation is the space decomposition. The decomposition result will directly affect the result of topological mapping, since it defines the basic units that are used to construct a topological region. There are several methods regarding to the space decomposition used in the computer vision society, such as Quadtrees, K-D trees, Octrees etc. Considering the repeatability and extendability, we perform the topological segmentation based on the decomposition result from a Quadtree structure in this paper.

Among the state-of-art works, spectral clustering [10] has been widely used in topological mapping. [11] takes key positions during the metric mapping process, and builds the adjacent matrix based on the distances between these key positions, then used spectral clustering to segment the existing in an online manner. The authors of [7] used spectral clustering as a off-line method on the global map. That work was followed by an incremental mapping and classification method presented[12]. In spite of the wide utilization, the spectral clustering method requires the calculation of inverting a big matrix (the size depends on the number of nodes in the graph), which makes it quite expensive in computation. Besides, Nadler et al in [13] stated that the spectral clustering has it's 'fundamental' limitations, e.g. the scalable problem. In this paper, we will provide an exemplar map following the idea from Nadler, to show that the spectral clustering wouldn't have a rational result out of it. Nevertheless, as a side-contribution to the community, we will also propose a approach based on spectral clustering in Sec. II, using a simple distance function to construct the adjacent matrix.

Considering the drawbacks of the spectral clustering method, we propose a new topological segmentation method based on *Shannon information theory*, using mutual information as the indicator of the relation between two adjacent nodes. This work is stimulated by the usage of mutual information in computer vision society, such as [14]. There are no direct references related to the topological mapping using information theory, to our bounded knowledge. The most relevant work is the active search scenario introduced by Davison [15]. Margarita et al extended his work to inferring of the hierarchical structure of visual maps. In this paper, we will give the definition of the mutual information used in the decomposition space. To segment the graph-like mutual information tree, a common approach is to use Chow-Liu tree to build the spanning tree, since Chow and Liu showed in their 1968 paper [16] that a full, joint probability probability density function can be optimally approximated as a product of second-order conditionals and marginal distributions, which will minimise the difference in Kullback-Leibler divergence [17]. The segmentation of the tree can then be done by selection of the level to which the Chow-Liu tree expands.

There are also other techniques, such as the efficient segmentation introduced by Mezaris et al in [18], can be used in the segmentation of a map. According to our test, it has issues with repeatability, and it's sensitive to parameter changes. Regarding to topological segmentation problem, it may not be a effective solution. The result will not be included in this paper.

We would like to stress the following possible contributions of this paper:

- A simple method to create the adjacent matrix for using spectral clustering in topological segmentation
- We point out there are some inherent limitations of spectral clustering in map segmentation, which is stimulated by Nadler's work [13].
- In spite of several existing works in computer vision society are using mutual information to do image segmentation and matching, we first proposed a topological mapping approach based on information theory.

As a detailed explanation of the introduction, the remainder of this paper is organized as follows: in the next section the spectral clustering approach and its limitations in topological mapping will be discussed. In Sec. III, we will explain our representation of the map based on information theory in detail. Then we will introduce the factorization and segmentation of the tree with different methods. The test results and conclusion will be given in the end.

II. Segmentation based on Spectral clustering and its Limitation

Usually, the raw map that we achieve from SLAM algorithm is in the form of occupancy grids. There are commonly three states of the occupancy: unknown, occupied and free space. To get a topological segmentation of the map is equivalent to the segmentation of the free space. In order to simplify the problem, we simulated a typical indoor map as shown in Fig. 1(a). In the purpose of getting a more general representation, a space decomposition method is firstly performed on the raw map. In this work we choose Quadtree as the basic strategy in space decomposition. It requires a input parameter to define to which maximum depth we build the tree. The deeper depth parameter means the more precise segmentation of the metric map. Each cell in the decomposition result is called a 'node' in the remainder of this paper.

The general algorithm of spectral clustering requires the neighbourhood graph together with the corresponding *adjacency matrix* W with $n \times n$ elements $W(i,j) = \omega_{ij}$, where n is the number of nodes in the graph. For various methods in this area, a big difference is the definition of the weight ω_{ij} between nodes. In our work, we define the weight for two nodes as follows:

$$\omega_{ij} = \begin{cases} l_{co}(i,j), & \text{if i and j are adjacent} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

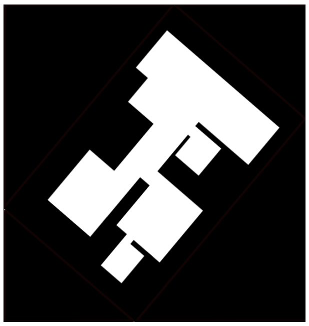
where $l_{co}(i,j)$ is the length of the common edge between the node i and j. Following the notation of von Luxburg in [10], the algorithm of spectral clustering is shown in Alg. 1

Algorithm 1: Spectral Clustering on Topological Segmentation

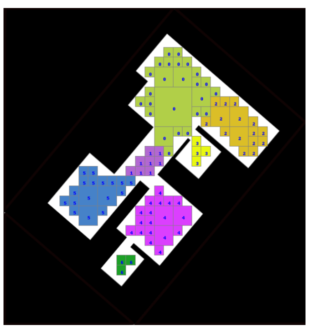
Input:
Adjacency matrix W, where $W(i,j)$ indicates the weight between two nodes i and j;
The request number of clusters k
$T = \{T_1, T_2, \dots T_k\} \simeq \{1, 2, \dots, k\}$
Output:
The list of indices corresponding to the nodes,
1 Calculate the normalized graph Laplacians using $L_{sym} := I - D^{-1/2} W D^{-1/2}$ or $L_{rw} := I - D^{-1} W$, where $D = \mathrm{diag} d_1, \dots, d_n$ and $d_i = \sum_{j=1}^{n} \omega_{ij}$;
2 Calculate the k smallest eigenvectors $u_1, \dots, u_k$ of L (either L_{sym} or L_{rw}), form the matrix $U = [u_1 \dots u_k] \in \mathbb{R}^{n \times k}$
3 Set $\hat{U}$ to be U with rows re-normalized to have unit norm, by $\hat{U}_{ij} = \frac{U_{ij}}{(\sum_j U_{ij})^{\frac{1}{2}}}$
4 Use k-means clustering on the rows of $\hat{U}$, into k clusters
5 Assign label T_i to cluster j if and only if row j of $\hat{U}$ is assigned to cluster j.

A. Sensitivity to k

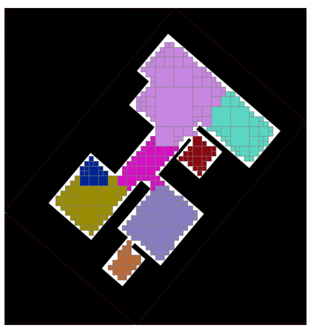
The decomposition with different level and results for the topological segmentation on the simulated environment are shown in Fig. 1(b) and Fig. 1(c). We could see from the results that two factors will affect the results much. First is the decomposition level, i.e. sometimes the shallower level will naturally divide the separated regions directly, such as the dark green cluster number 6 in Fig. 1(b). The second factor is the number of clusters. We intendedly choose 8 as the number of clusters for the test in Fig. 1(c). A redundant cluster (marked with dark blue) is detected without obvious reasons.



(a) Raw map



(b) Segmentation Result at decomposition level 6



(c) Segmentation Result at decomposition level 7

Fig. 1. Segmentation result based on spectral clustering

This can be considered as the first limitation of spectral clustering method, while most of applications we have neither information on the number of regions in a global map, nor rough ideas of the optimal size of the cells.

B. Unrepeatable results

Another result of spectral clustering is achieved by processing the map provided by Choi et al in [7]. The result is shown in Fig. 3. We show our result in Fig. 3(b). By comparing with the result from [7] in Fig. 3(a), we think these results are both acceptable in the general sense of "separating rooms". However, with the same parameters, sometimes we can also achieve other results such as Fig. 3(c), this segmentation somehow make sense from the raw sensory data aspect, but less useful for environment modelling e.g. semantic mapping and reasoning. These results imply the instability of the similar approach. The reason can be partially inferred from the essence of the spectral clustering.

C. Reasoning

The eigenvalues and first 3 elements in eigenvectors are shown in Fig. 2. [1] It plots the eigenvalues of the adjacency matrix of Choi's map, as well as the eigenvectors corresponding to the second to the fourth eigenvalues. By focusing on the region marked within the red rectangle, we could observe the following fact: in order to achieve a better segmentation of the eigenvectors, it requires more eigenvectors to be taken into account. If k is not properly selected, the similarities of the eigenvectors in lower dimension directly cause the unstableness of the k-means segmentation results.

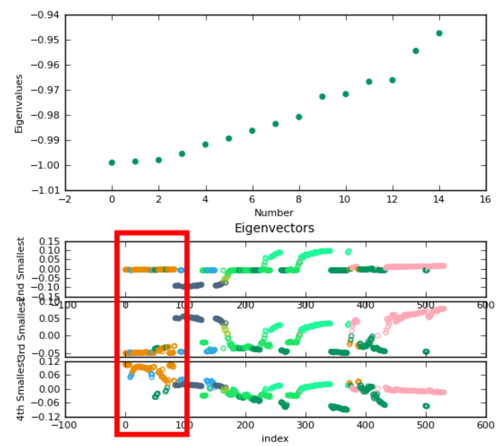


Fig. 2. The plot of the eigenvalues and eigenvectors of the Laplacians Spectrum of the test map at decomposition level 6

D. An extreme case

Moreover, under extreme cases, the spectral clustering can not even rationally work. Following the point cloud example provided in [13], we create a simulated environment as shown in Fig. 4(a). The environment is constructed by a rectangle 'corridor' area and circler 'room'. The proposed spectral clustering will achieve a result as shown in 4(b), other spectral clustering method with different weighting functions provide similar results: the 'room' and the 'corridor' can not be correctly segmented. Then we increased the k, hoping that the segmentation can randomly hit the 'room'. The result of k=3 is shown in Fig. 4(c). It still can not find the rational segmentation. This phenomenon was called the "scaling problem" of the spectral clustering in [13]. It means that we can not simply increase the k to approach the result that we are looking for. The bad news is that actually there is no feasible ways to control the segmentation result of spectral clustering.

Beside the listed drawbacks, there are other issues such as it is hard to combine several sub-maps to a compact union, while the tree structure of each segmented region is not clear.

[1]The smallest eigenvalue indicates the connectivity of the graph, therefore it is not shown here.

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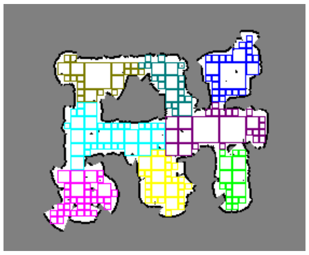
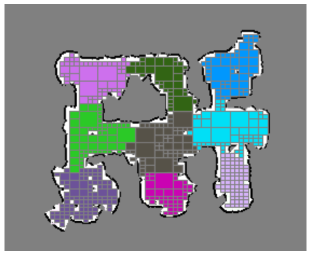
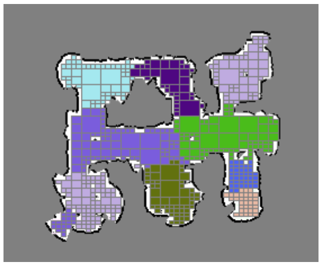


(a) Choi's result

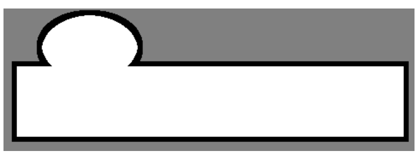


(b) Our result

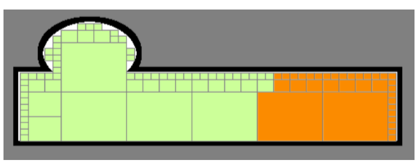


(c) Instability of the segmentation

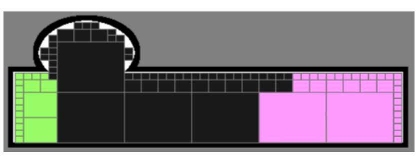
Fig. 3. Comparison of spectral clustering with different distance functions



(a)



(b) Result with k=2



(c) Result with k=3

Fig. 4. An example map showing the limitation of spectral clustering

All in all, a more sophisticated method is required to tackle the topological segmentation problem. We will introduce our advanced approach in the following section.

III. Mutual Information Tree and Factorization

Taking the example depicted in Fig. 1, we can observe that the segmentation is a clustering method where typical clusters are formed by some center cells surrounded by supporter cells. We propose a segmentation method based on the idea that if two neighboring cells have a lot of supporters in common, then the link between them is strong. As exposed above, the supporters are weighted according to the length of the common edge.

To achieve that goal, we use Shannon's information theory to represent the relationship between cells, via the computation of the entropy divergence between the supports of neighboring cells.

Using the notation of Mackay [19] and Davison [15], with the knowledge that the direct neighbours of a center cell x lies within the discrete 'alphabet' $A_x = \{a_1, a_2, \dots\}$ of cells labels, we introduce the probability distribution $P(N \mid x)$ of the supporting strength of the neighbors n of x to the cell x proportional to the length of the common edge:

$$P(n \mid x) = \frac{l_{co}(n,x)}{\sum_{x \in A_x} l_{co}(n,x)}. \tag{2}$$

Similarly, we introduce the support of a cell n to the link between 2 cells x and y as proportionnal to the product of the length of the common edge with each of the 2 cells:

$$P(n \mid x,\ y) = \frac{l_{co}(n,x) \times l_{co}(n,y)}{\sum_{n \in A_{xy}} l_{co}(n,x) l_{co}(n,y)}. \tag{3}$$

Notice that this expression is symmetric in X and Y and is 0 if the cell n is not a neighbor of both x and y. Therefore these expressions need only be computed locally.

The *information entropy* $H(N \mid x)$ of the distribution $P(N \mid x)$ is:

$$H(N \mid x) = E\left[\log_2 \frac{1}{P(N \mid x)}\right] = \sum_{n \in A_x} P(n \mid x) \log_2 \frac{1}{P(n \mid x)}. \tag{4}$$

In a similar way,

$$H(N \mid x,y) = E\left[\log_2 \frac{1}{P(N \mid x,y)}\right] = \sum_{n \in A_x} P(n \mid x,y) \log_2 \frac{1}{P(n \mid x,y)}. \tag{5}$$

And the *mutual information* $I(N \mid x; N \mid x,\ y)$, noted $I(x,y)$ in the rest of the text, between the distributions $P(N \mid X)$ and $P(N \mid X,\ Y)$ is:

$$I(x,y) = H(N \mid x) - H(N \mid x,\ y) \tag{6}$$

Following the expression in [15], it indicates the average expected reduction in entropy in one cell on learning the supporting status of one of its neighbours. Therefore, $I(x;y)$ indicates how much exclusive support can one cell X get, considering his neighbour Y. When this number is big, it means the connections between X and Y are close, in the sense of forming a compact cluster.

Based on this measure, we can then define a *mutual information* matrix as in Eq. 7, where N is the total number of the nodes in the map.

$$I = \begin{pmatrix} * & I(x_1,x_2) & \dots & I(x_1,x_N) \\ I(x_2,x_1) & * & \dots & I(x_2,x_N) \\ \vdots & \vdots & \vdots & \vdots \\ I(x_N,x_1) & I(x_N,x_2) & \dots & * \end{pmatrix} \tag{7}$$

The matrix is symmetric and the diagonal elements are meaningless while they represents the connection weight between a node and itself.

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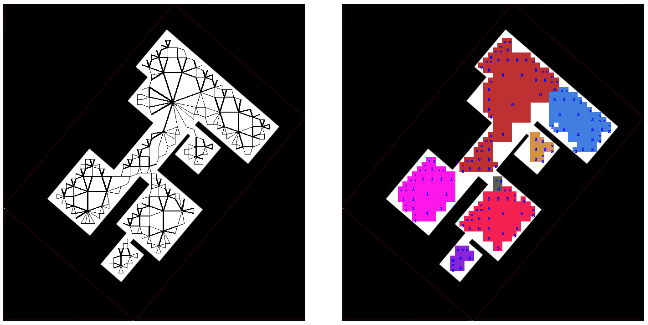
The plot of *mutual information* for the simulated environment Fig. 1(a) is shown in Fig. 5(a)



(a) The plot of *mutual information*. The thickness of edges represent the magnitude of *mutual information*

(b) Segmentation result at $th = 2.0$

Fig. 5. The representation of the *mutual information* graph and the segmentation result.

IV. Tree Factorization and Chow-Liu Tree

The *mutual information* graph we got in the previous section is a non-spanning graph. There are cycles in the graph structure. To generate a spanning tree from the graph, the usual way is to use Chow-Liu tree factorization. The essence of Chow-Liu tree is a *Maximum Weights Spanning Tree*. We build the Chow-Liu tree by following the standard procedure, using the *mutual information* as the weights of edges. The tree-building starts from the maximum weighted edges. At each step, the edge with the biggest weight will be added to the tree, if and only if the edge won't forming any cycles with the existing edges. This process will stop when all the nodes are connected, returning a full connected Chow-Liu tree.

As for the application of Chow-Liu tree on a real dataset. The *mutual information* graph is not likely to be a full connected tree. We need to separate the un-connected sub-graphs first. The algorithm can be stated as follows (Alg. 2):

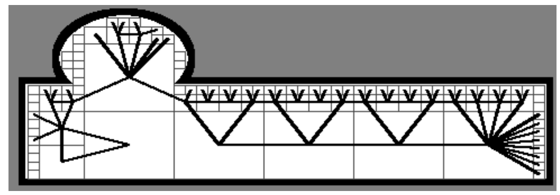
Algorithm 2: Search all sub-graphs in the graph

Input:
The set of all the nodes in the graph: $Ni = 0$
Output:
Separated sets of connected nodes: N_i

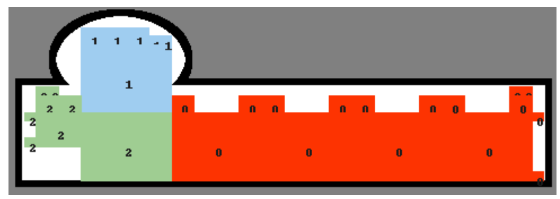
while $N! = \phi$ **do**
- Define a random node $n_i \in N$ as a seed node
- Find all the nodes that connected to the seed node using Breadth First Search, and save them as set N_i
- $N = N \backslash N_i$, remove the N_i subset from N
- $i = i + 1$

For each N_i, at least one region should be kept. In other cases, the factorization process will stop by a certain threshold level, i.e. the weak links whose weights are under the pre-defined threshold level will be ignored.

As for another example, we build the spanning tree over the extreme case, on which the spectral clustering fails. The segmented result is shown in Fig. 5(b). As a natural *mutual information* graph, we could see that the free space in the map is segmented into 3 parts. If we ignore the weak weighted edges under 2.0, the result is shown in 6(b). Although it can not give the exactly expected segmentation as a rectangle plus a eclipse, the 3-node structure is reasonable in the sense of regional detection.



(a) *mutual information* graph



(b) Segmentation result at $th = 2.0$

Fig. 6. Segmentation result on the extreme case

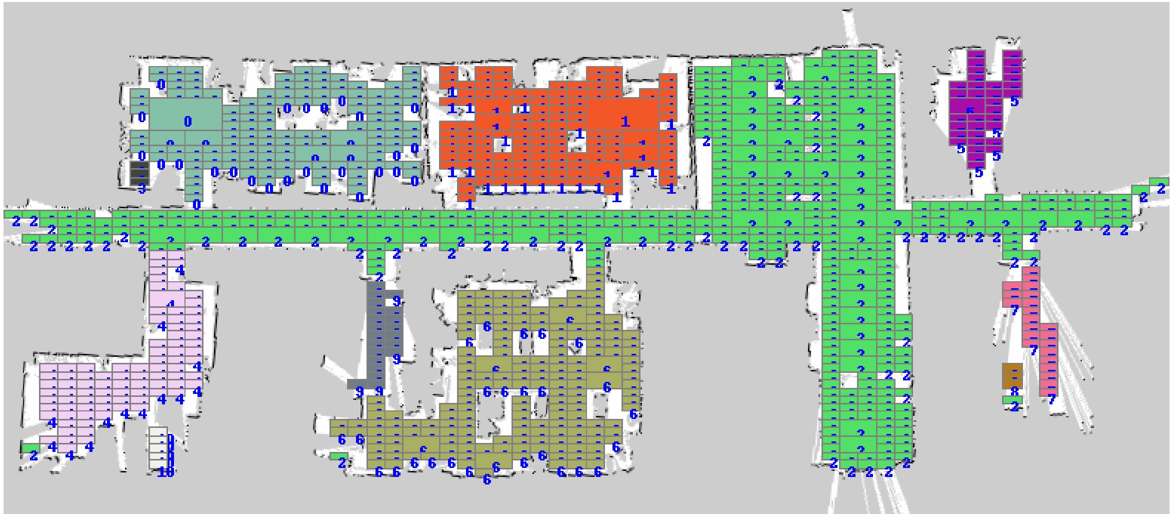
V. Results on Metric Maps

A. Functional Test

In order to test our method in a larger scale with real data input, we carried out a test on the occupancy grid map created in an office environment. The map covers a area of 50×18 square meters. The structure of the environment is quite complex, including doorways, corridor and office rooms etc. The segmentation results are shown in Fig. 7.

Actually, with a low threshold level such as 1.0, the proposed method can get roughly exactly the same result as the spectral clustering based method (the one shown in Fig. 7(a)). In the matter of space, the result is not shown in the paper. The interesting story occurs when want to do more than that. While the occupancy map is too big in the sense of semantic mapping, i.e. one whole office room is too hasty for the reasoning of the environment. Take the region number 6 (light brown) in Fig. 7(a) as an example, there are separated parts inside the room. How could we do a further segmentation on that? As for spectral clustering, Fig. 4(b) and (c) have already given the answers that the spectral clustering doesn't have the ability to provide the rational segmentation the environment at a more precise level based on the number of k. This is again due to the 'scale' problem which is coming together with its nature. On the contrary, the proposed method based on the *mutual information* graph is able to achieve that as shown in Fig. 7(b).

As a direct extension of the proposed method, by combining with the Alg. 2, the Chow-Liu tree can be pruned inside each



(a) Segmentation by optimized spectral clustering

(b) Segmentation by mutual information at $th = 1.6$. The noisy small regions can be eliminated by median filters

Fig. 7. Segmentation of a metric map

sub-graph. This feature is supposed to be the most important advantage comparing to spectral clustering. One problem may be the noisy cuts in the graph, as shown in the left bottom part in Fig. 7(b).

In the matter of fact, there are small regions which can be considered as segmentation noise in both cases. These noisy regions can be eliminated by median filters. This assumption can be inferred from the plot of the *mutual information* matrix I, as shown in Fig. 8. To simplify the expressions, we use the simulated map in Fig. 4 as an example. We can see that most of the positive weights are lying on the diagonal of the I matrix. The indices of the elements in the matrix are roughly following the nearest-neighbour search. Therefore, it indicates that most of the 'support' are from the neighbours, which fits our assumption in the definition of the *mutual information*. This plot also implies the potential 'centers' of each cluster. If we accumulated the weights values that lie in the same row or column, the result represents the impact of the certain node when a cluster is generated around it. For example, in the Fig. 8, we can intuitively see three dominating 'center' nodes, which leads to three potential cuts of the whole region. The *mutual information* matrix for 7 shows the similar shape, but with ambiguous dominant 'center' nodes because of the size of the matrix.

Combining the tree structure and the *mutual information* matrix, the result from *mutual information* graph can be easily controlled by selecting different threshold level, since the higher threshold level will provide more regions and the strong links come together with high accumulated weights. On the contrary, the result of spectral clustering is not intuitively controllable; once the user requires more sub-regions, the *k*-means clustering needs to be run since the beginning, which requires a sequence of test-and-run processes.

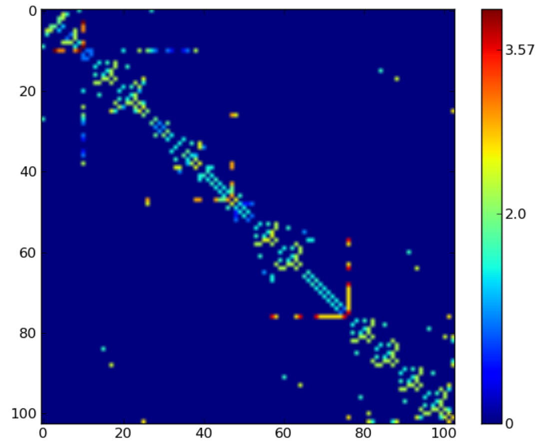


Fig. 8. The plot of the *mutual information* matrix I of the map in Fig. 4

B. Computational Complexity

The time consumption of the primary steps is shown in Fig. 9. This test uses both methods, *mutual information* graph and spectral clustering to segment two different maps separately. In the purpose of evaluating the computational complexity, the two maps have different number of nodes, due to the scale of the map. In the procedure of both methods, the Quadtree works as the basic space decomposition method. The time consumption is proportional to the maximum depth of the tree with the computational complexity of $O(n+1)^2$), where n is the maximum depth of the tree [2]. This decomposition process is not closely related to the clustering methods, so that the calculation of the time consumption is started after that.

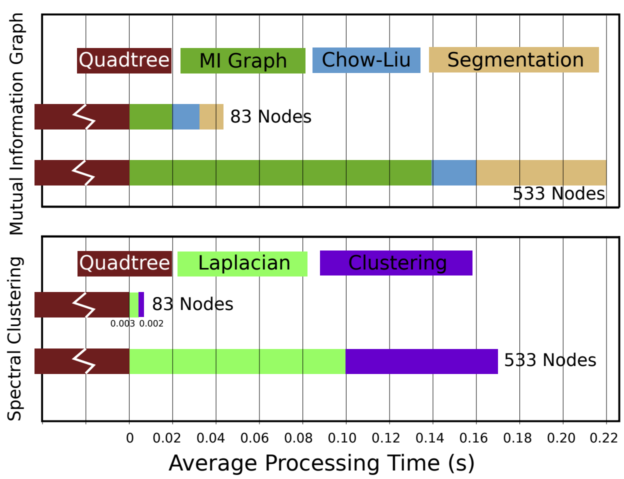


Fig. 9. A comparison of the computational complexity

[2]A typical processing time for Quadtree decomposition on a laptop with 2.5GHz CPU is around 1.4 seconds for the depth of 7

Regarding to optimizing the computational complexity, we did some pre-processing to get the adjacent matrix. We calculate all the neighbour relationships separately and save them in an indexed table. Therefore, for the *mutual information* calculation in Eq. 6, instead of $O(n^2)$ complexity (for n nodes searching neighbours among n nodes), the process only takes $O(n)$ complexity. The result in Fig. 9 confirmed this assumption, where the second map has around 7 times more nodes comparing to the first map (533:83). We can see that the "MI Graph" (short for *mutual information* graph, in dark green) process consumes roughly 7 times more time for the second map.

Comparing to the proposed method, the spectral clustering method is much faster when the graph is small. However, as the matrix inversion is the major step in the calculation of the Laplacians, which holds $O(n^3)$ complexity, the processing time dramatically rises when it comes to the second map.

According to the result in Fig. 9, our method shows an important timing features. This 'lightweighted' characteristic is helpful and efficient in the real-time mobile robot mapping and navigation applications.

VI. Extension of the Method

The segmentation result is based on the Quadtree decomposition of the free space. Although it can give a corresponding representation of the environment, it's still not convenient to handle one cluster with a list of nodes. Therefore, we also developed an algorithm to extract polygon representations from the segmentation results. The method extracts convex polygons from subgrapgs based on the further segmentation inside each regions, then merge the convex polygons to possibly non-convex shapes. Typical results is shown in Fig. 10.

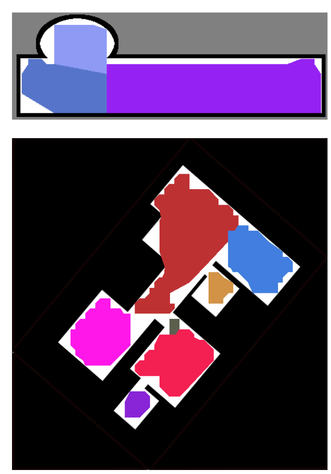


Fig. 10. Polygon representations of the segmentation result

The result can be represented by a number of labels and corresponding lists which are consist of the vertices of the polygons. Comparing to other results in this paper, this representation is more compact in storage consumption and more intuitive for a human user. Moreover, during the process of the polygon generation, the noisy gaps and isolated single leaves

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can be properly removed. All these advantages will lead to a smoother representation of the map. The detailed discussion and analysis will be given in our further reports.

VII. Conclusion

In this paper, we first gave a new segmentation method based on the widely cited spectral clustering theory. By comparing our results with others', we show the inevitable drawbacks of topological segmentation methods which are based on spectral clustering. Then we proposed a segmentation method based on *information theory*. This method is based on the *mutual information* graph and Chow-Liu tree factorization. We stated and certified the following characteristics of the proposed method:

1) Fully Scalable. The segmentation result can be easily optimized by thresholding on local sub-graphs.
2) Stable segmentation results. The Chow-Liu tree factorization is unique under the same weight function, which avoids the randomness introduced by *k*-means clustering method, which is used in spectral clustering.
3) Computational cheap in large scale. We have showed that the method can work under $O(n)$ complexity, on the conditional that the I matrix is fully calculated.

While the proposed method is based on *mutual information* between neighbours, it allows us to extend the method easily by fusing other sensory information, such as vision. These information will greatly increase the robustness of the segmentation and towards a more rational topological/semantic map. Another potential future work is to perform the method in an incremental manner. Since the environment can be presented in an undirected graph structure, it is likely to extend the links with several neighbour sub-maps. The testing results will be presented after our further researches.

References

[1] C. Galindo, A. Saffiotti, S. Coradeschi, P. Buschka, J. Fernandez-Madrigal, and J. Gonzalez, "Multi-hierarchical semantic maps for mobile robotics," aug. 2005, pp. 2278 – 2283.

[2] O. M. Mozos, A. Rottmann, R. Triebel, P. Jensfelt, and W. Burgard, "Supervised semantic labeling of places using information extracted from sensor data," *Robotics and Autonomous Systems*, vol. 55, pp. 391–402, 2007.

[3] S. Vasudevan and S. G "Cognitive maps for mobile robots–an object based approach," *Robotics and Autonomous Systems*, vol. 55, no. 5, pp. 359 – 371, 2007, from Sensors to Human Spatial Concepts. [Online]. Available: http://www.sciencedirect.com/science/article/B6V16-4MY0MK7-1/2/e379fd59a33b6d0a42355ba120c444e9

[4] A. Tapus and R. Siegwart, "Incremental robot mapping with fingerprints of places," in *Intelligent Robots and Systems, 2005.(IROS 2005). 2005 IEEE/RSJ International Conference on*, 2005, pp. 2429–2434.

[5] M. Liu, R. S. Davide Scaramuzza, Cédric Pradalier, and Q. Chen, "Scene recognition with omnidirectional vision for topological map using lightweight adaptive descriptors," in *IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS 2009*, 2009.

[6] S. Simhon and G. Dudek, "A global topological map formed by local metric maps," in *IEEE/RSJ International Conference on Intelligent Robotic Systems*, vol. 3. Citeseer, 1998, pp. 1708–1714.

[7] J. Choi, M. Choi, and W. K. Chung, "Incremental topological modeling using sonar gridmap in home environment," pp. 3582–3587, Oct. 2009. [Online]. Available: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=5354247

[8] S. Thrun, "Learning metric-topological maps for indoor mobile robot navigation," *Artificial Intelligence*, vol. 99, no. 1, pp. 21 – 71, 1998. [Online]. Available: http://www.sciencedirect.com/science/article/B6TYF-3T0XP9D-2/2/c696e28377beaa9998c9a067cb030cbb

[9] H. Choset, "Sensor based motion planning: The hierarchical generalized Voronoi graph," Ph.D. dissertation, Citeseer, 1996.

[10] U. V. Luxburg, "A tutorial on spectral clustering," *Statistics and Computing*, no. August 2006, pp. 395–416, 2007.

[11] E. Brunskill, T. Kollar, and N. Roy, "Topological Mapping Using Spectral Clustering and Classification," in *International Conference on Intelligent Robots and Systems*. IEEE, 2007, pp. 3491–3496.

[12] J. Choi, M. Choi, and W. K. Chung, "Incremental topological modeling using sonar gridmap in home environment," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*. Ieee, Oct. 2009, pp. 3582–3587. [Online]. Available: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=5354247

[13] B. Nadler and M. Galun, "Fundamental limitations of spectral clustering," *Advances in Neural Information Processing Systems*, vol. 19, p. 1017, 2007.

[14] J. Kim, J. Fisher III, A. Yezzi, M. Çetin, and A. Willsky, "A nonparametric statistical method for image segmentation using information theory and curve evolution," *IEEE Transactions on Image Processing*, vol. 14, no. 10, 2005.

[15] A. J. Davison, "Active Search for Real-Time Vision," in *ICCV*, 2005, pp. 66–73 Vol.1.

[16] C. Chow and C. Liu, "Approximating discrete probability distributions with dependence trees," *IEEE transactions on Information Theory*, vol. 14, no. 3, pp. 462–467, 1968.

[17] S. Kullback and R. Leibler, "On information and sufficiency," *The Annals of Mathematical Statistics*, vol. 22, no. 1, pp. 79–86, 1951.

[18] V. Mezaris, I. Kompatsiaris, and M. Strintzis, "A framework for the efficient segmentation of large-format color images," in *Proc. International Conference on Image Processing*, vol. 1. Citeseer, 2002, pp. 761–764.

[19] D. MacKay, *Information theory, inference, and learning algorithms*. Cambridge Univ Pr, 2003.

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